(ii) For right-tailed test, the p-value is the area to the right of the calculated value of the test statistic. For instance, if $z_{cal} = +2.00$, then the area to the right of it is 0.5000 – 0.4772 = 0.0228, or the p value is 2.28 per cent.

Thus the decision rules for left-tailed test and right-tailed test are as under.

- Reject H_0 if p-value $\leq \alpha$
- Accept H_0 if p-value $> \alpha$

(iii) For a two-tailed test, the p-value is twice the tail area. If the calculated value of the test statistic falls in the left tail (or right tail), then the area to the left (or right) of the calculated value is multiplied by 2.

Example 10.6: An auto company decided to introduce a new six cylinder car whose mean petrol consumption is claimed to be lower than that of the existing auto engine. It was found that the mean petrol consumption for 50 cars was 10 km per litre with a standard deviation of 3.5 km per litre. Test for the company at 5 per cent level of significance, the claim that in the new car petrol consumption is 9.5 km per litre on the average. *[HP Univ., MBA, 1989]*

Solution: Let us assume the null hypothesis H_0 that there is no significant difference between the company's claim and the sample average value, that is,

$$H_0 : \mu = 9.5 \text{ km/litre and } H_1 : \mu \neq 9.5 \text{ km/litre}$$

Given $\bar{x} = 10$, $n = 50$, $s = 3.5$, and $z_{\alpha/2} = 1.96$ at $\alpha = 0.05$ level of significance. Thus using the z-test statistic

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{10 - 9.5}{3.5/\sqrt{50}} = 1.010$$

Since $z_{cal} = 1.010$ is less than its critical value $z_{\alpha/2} = 1.96$ at $\alpha = 0.05$ level of significance, the null hypothesis is accepted. Hence we can conclude that the new car's petrol consumption is 9.5 km/litre.

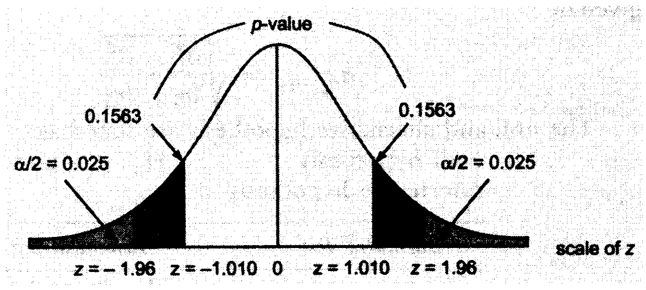

Figure 10.8

The p-value approach The null hypothesis accepted H_0 because $z_{cal} = 1.010$ lies in the acceptance region. The probability of finding $z_{cal} = 1.010$ or more is 0.3437 (from normal table). The p-value is the area to the right as well as left of the calculated value of z-test statistic (for two-tailed test). Since $z_{cal} = 1.010$, then the area to its right is 0.5000 – 0.3437 = 0.1563 as shown in Fig. 10.8.

Since it is the two-tailed test, p-value becomes 2(0.1563)=0.3126. Since 0.3126 > α=0.05, null hypothesis H_0 is accepted.

10.7.4 Hypothesis Testing for Difference between Two Population Means

If we have two independent populations each having its mean and standard deviation as:

Population	Mean	Standard Deviation
1	μ_1	σ_1
2	μ_2	σ_2

then we can extend the hypothesis testing concepts developed in the previous section to test whether there is any significant difference between the means of these populations.

Let two independent random samples of large size n_1 and n_2 be drawn from the first and second population, respectively. Let the sample means so calculated be $\bar{x}_1$ and $\bar{x}_2$.

The z-test statistic used to determine the difference between the population means $(\mu_1 - \mu_2)$ is based on the difference between the sample means $(\bar{x}_1 - \bar{x}_2)$ because sampling distribution of $\bar{x}_1 - \bar{x}_2$ has the property $E(\bar{x}_1 - \bar{x}_2) = (\mu_1 - \mu_2)$. This test statistic will follow the standard normal distribution for a large sample due to the central limit theorem. The z-test statistic is

$$\text{Test statistic: } z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma_{\bar{x}_1 - \bar{x}_2}} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

where $\sigma_{\bar{x}_1 - \bar{x}_2}$ = standard error of the statistic $(\bar{x}_1 - \bar{x}_2)$

$\bar{x}_1 - \bar{x}_2$ = difference between two sample means, that is, sample statistic

$\mu_1 - \mu_2$ = difference between population means, that is, hypothesized population parameter

If $\sigma_1^2 = \sigma_2^2$, the above formula algebrically reduces to:

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$$

If the standard deviations σ_1 and σ_2 of each of the populations are *not known*, then we may estimate the standard error of sampling distribution of the sample statistic $\bar{x}_1 - \bar{x}_2$ by substituting the sample standard deviations s_1 and s_2 as estimates of the population standard deviations. Under this condition, the standard error of $\bar{x}_1 - \bar{x}_2$ is estimated as:

$$s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

The standard error of the *difference between standard deviation of sampling distribution* is given by

$$\sigma_{\sigma_1 - \sigma_2} = \sqrt{\frac{\sigma_1^2}{2n_1} + \frac{\sigma_2^2}{2n_2}}$$

The null and alternative hypothesis are stated as:

Null hypothesis : $H_0 : \mu_1 - \mu_2 = d_0$

Alternative hypothesis :

One-tailed Test	*Two-tailed Test*
$H_1 : (\mu_1 - \mu_2) > d_0$	$H_1 : (\mu_1 - \mu_2) \neq d_0$
$H_1 : (\mu_1 - \mu_2) < d_0$	

where d_0 is some specified difference that is desired to be tested. If there is no difference between μ_1 and μ_2, i.e. $\mu_1 = \mu_2$, then $d_0 = 0$.

Decision rule: Reject H_0 at a specified level of significance α when

One-tailed test	*Two-tailed test*
• $z_{cal} > z_\alpha$ [or $z < -z_\alpha$ when $H_1 : \mu_1 - \mu_2 < d_0$	• $z_{cal} > z_{\alpha/2}$ or $z_{cal} < -z_{\alpha/2}$
• When p-value $< \alpha$	

Example 10.7: A firm believes that the tyres produced by process A on an average last longer than tyres produced by process B. To test this belief, random samples of tyres produced by the two processes were tested and the results are:

Process	*Sample Size*	*Average Lifetime (in km)*	*Standard Deviation (in km)*
A	50	22,400	1000
B	50	21,800	1000

Is there evidence at a 5 per cent level of significance that the firm is correct in its belief?

Solution: Let us take the null hypothesis that there is no significant difference in the average life of tyres produced by processes A and B, that is,

$$H_0 : \mu_1 = \mu_2 \text{ or } \mu_1 - \mu_2 = 0 \text{ and } H_1 : \mu_1 \neq \mu_2$$

Given, $\bar{x}_1 = 22{,}400$ km, $\bar{x}_2 = 21{,}800$ km, $\sigma_1 = \sigma_2 = 1000$ km, and $n_1 = n_2 = 50$. Thus using the z-test statistic

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma_{\bar{x}_1 - \bar{x}_2}} = \frac{\bar{x}_1 - \bar{x}_2}{\sigma_{\bar{x}_1 - \bar{x}_2}} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

$$= \frac{22{,}400 - 21{,}800}{\sqrt{\dfrac{(1000)^2}{50} + \dfrac{(1000)^2}{50}}} = \frac{600}{\sqrt{20{,}000 + 20{,}000}} = \frac{600}{200} = 3$$

Since the calculated value $z_{cal} = 3$ is more than its critical value $z_{\alpha/2} = \pm 1.645$ at $\alpha = 0.05$ level of significance, therefore H_0 is rejected. Hence we can conclude that the tyres produced by process A last longer than those produced by process B.

The *p*-value approach:

$$p\text{-value} = P(z > 3.00) + P(z < -3.00) = 2\,P(z > 3.00)$$
$$= 2(0.5000 - 0.4987) = 0.0026$$

Since p-value of 0.026 is less than specified significance level $\alpha = 0.05$, H_0 is rejected.

Example 10.8: An experiment was conducted to compare the mean time in days required to recover from a common cold for person given daily dose of 4 mg of vitamin C versus those who were not given a vitamin supplement. Suppose that 35 adults were randomly selected for each treatment category and that the mean recovery times and standard deviations for the two groups were as follows:

	Vitamin C	*No Vitamin Supplement*
Sample size	35	35
Sample mean	5.8	6.9
Sample standard deviation	1.2	2.9

Test the hypothesis that the use of vitamine C reduces the mean time required to recover from a common cold and its complications, at the level of significance $\alpha = 0.05$.

Solution: Let us take the null hypothesis that the use of vitamin C reduces the mean time required to recover from the common cold, that is

$$H_0 : (\mu_1 - \mu_2) \leq 0 \text{ and } H_1 : (\mu_1 - \mu_2) > 0$$

Given $n_1 = 35$, $\bar{x}_1 = 5.8$, $s_1 = 1.2$ and $n_2 = 35$, $\bar{x}_2 = 6.9$, $s_2 = 2.9$. The level of significance, $\alpha = 0.05$. Substituting these values into the formula for z-test statistic, we get

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma_{\bar{x}_1 - \bar{x}_2}} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

$$= \frac{5.8 - 6.9}{\sqrt{\dfrac{(1.2)^2}{35} + \dfrac{(2.9)^2}{35}}} = \frac{-1.1}{\sqrt{0.041 + 0.240}} = -\frac{1.1}{0.530} = -2.605$$

Using a one-tailed test with significance level $\alpha = 0.05$, the critical value is $z_\alpha = 1.645$. Since $z_{cal} < z_\alpha$ (= 1.645), the null hypothesis H_0 is rejected. Hence we can conclude that the use of vitamic C does not reduce the mean time required to recover from the common cold.

Example 10.9: The Educational Testing Service conduced a study to investigate difference between the scores of female and male students on the Mathematics Aptitude Test. The study identified a random sample of 562 female and 852 male students who had achieved the same high score on the mathematics portion of the test. That is, the female and male students viewed as having similar high ability in mathematics. The verbal scores for the two samples are given below:

	Female	*Male*
Sample mean	547	525
Sample stundard deviation	83	78

Do the data support the conclusion that given populations of female and male students with similar high ability in mathematics, the female students will have a significantly high verbal ability? Test at $\alpha = 0.05$ significance level. What is your conclusion?

[*Delhi Univ., MBA, 2003*]

Solution: Let us take the null hypothesis that the female students have high level verbal ability, that is,

$$H_0 : (\mu_1 - \mu_2) \geq 0 \text{ and } H_1 : (\mu_1 - \mu_2) < 0$$

Given, for female students: $n_1 = 562$, $\bar{x}_1 = 547, s_1 = 83$, for male students: $n_2 = 852$, $\bar{x}_2 = 525, s_2 = 78$, and $\alpha = 0.05$.

Substituting these values into the z-test statistic, we get

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \frac{547 - 525}{\sqrt{\dfrac{(83)^2}{562} + \dfrac{(78)^2}{852}}}$$

$$= \frac{22}{\sqrt{12.258 + 7.140}} = \frac{22}{\sqrt{19.398}} = \frac{22}{4.404} = 4.995$$

Using a one-tailed test with α=0.05 significance level, the critical value of z-test statistic is $z_\alpha = \pm\ 1.645$. Since $z_{cal} = 4.995$ is more than the critical value $z_\alpha = 1.645$, null hypothesis, H_0 is rejected. Hence, we conclude that there is no sufficient evidence to declare that difference between verbal ability of female and male students is significant

Example 10.10: In a sample of 1000, the mean is 17.5 and the standard deviation is 2.5. In another sample of 800, the mean is 18 and the standard deviation is 2.7. Assuming that the samples are independent, discuss whether the two samples could have come from a population which have the same standard deviation.

[*Saurashtra Univ., BCom, 1997*]

Solution: Let us take the hypothesis that there is no significant difference in the standard deviations of the two samples, that is, $H_0 : \sigma_1 = \sigma_2$ and $H_1 : \sigma_1 \neq \sigma_2$.

Given, $\sigma_1 = 2.5$, $n_1 = 1000$ and $\sigma_2 = 2.7$, $n_2 = 800$. Thus we have

$$\text{Standard error, } \sigma_{\sigma_1 - \sigma_2} = \sqrt{\frac{\sigma_1^2}{2n_1} + \frac{\sigma_2^2}{2n_1}}$$

$$= \sqrt{\frac{(2.5)^2}{2000} + \frac{(2.7)^2}{1600}} = \sqrt{\frac{6.25}{2000} + \frac{7.29}{1600}} = 0.0876$$

Applying the z-test statistic, we have

$$z = \frac{\sigma_1 - \sigma_2}{\sigma_{\sigma_1 - \sigma_2}} = \frac{2.7 - 2.5}{0.0876} = \frac{0.2}{0.0876} = 2.283$$

Since the $z_{cal} = 2.283$ is more than its critical value $z = 1.96$ at $\alpha = 5$ per cent, the null hypothesis H_0 is rejected. Hence we conclude that the two samples have not come from a population which has the same standard deviation.

Example 10.11: The mean production of wheat from a sample of 100 fields is 200 lbs per acre with a standard deviation of 10 lbs. Another sample of 150 fields gives the mean at 220 lbs per acre with a standard deviation of 12 lbs. Assuming the standard deviation of the universe as 11 lbs, find at 1 per cent level of significance, whether the two results are consistent. [*Punjab Univ., MCom, Mangalore MBA, 1996*]

Solution: Let us take the hypothesis that the two results are consistent, that is

$$H_0 : \sigma_1 = \sigma_2 \text{ and } H_1 : \sigma_1 \neq \sigma_2.$$

Given $\sigma_1 = \sigma_2 = 11, n_1=100, n_2=150$. Thus

$$\sigma_{\sigma_1 - \sigma_2} = \sqrt{\frac{\sigma^2}{2}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} = \sqrt{\frac{(11)^2}{2}\left(\frac{1}{100}+\frac{1}{150}\right)} = 1.004$$

Applying the z-test statistic we have

$$z = \frac{\sigma_1 - \sigma_2}{\sigma_{\sigma_1 - \sigma_2}} = \frac{10-12}{1.004} = -\frac{2}{1.004} = -1.992$$

Since the $z_{cal} = -1.992$ is more than its critical value $z = -2.58$ at $\alpha = 0.01$, the null hypothesis is accepted. Hence we conclude that the two results are likely to be consistent.

Self-Practice Problems 10A

10.1 The mean breaking strength of the cables supplied by a manufacturer is 1800 with a standard deviation of 100. By a new technique in the manufacturing process it is claimed that the breaking strength of the cables has increased. In order to test this claim a sample of 50 cables is tested. It is found that the mean breaking strength is 1850. Can we support the claim at a 0.01 level of significance?

10.2 A sample of 100 households in a village was taken and the average income was found to be Rs 628 per month with a standard deviation of Rs 60 per month. Find the standard error of mean and determine 99 per cent confidence limits within which the income of all the people in this village are expected to lie. Also test the claim that the average income was Rs 640 per month.

10.3 A random sample of boots worn by 40 combat soldiers in a desert region showed an average life of 1.08 years with a standard deviation of 0.05. Under the standard conditions, the boots are known to have an average life of 1.28 years. Is there reason to assert at a level of significance of 0.05 that use in the desert causes the mean life of such boots to decrease?

10.4 An ambulance service claims that it takes, on an average, 8.9 minutes to reach its destination in emergency calls. To check on this claim, the agency which licenses ambulance services had them timed on 50 emergency calls, getting a mean of 9.3 minutes with a standard deviation of 1.8 minutes. At the level of significance of 0.05, does this constitute evidence that the figure claimed is too low?

10.5 A sample of 100 tyres is taken from a lot. The mean life of the tyres is found to be 39,350 km with a standard deviation of 3260 km. Could the sample come from a population with mean life of 40,000 km? Establish 99 per cent confidence limits within which the mean life of the tyres is expected to lie.

[Delhi Univ., BA(H) Eco., 1996]

10.6 A simple sample of the heights of 6400 Englishmen has a mean of 67.85 inches and a standard deviation of 2.56 inches, while a simple sample of heights of 1600 Austrians has a mean of 68.55 inches and a standard deviation of 2.52 inches. Do the data indicate that the Austrians are on the average taller than the Englishmen? Give reasons for your answer.

[MD Univ., MCom, 1998; Kumaon Univ., MBA, 1999]

10.7 A man buys 50 electric bulbs of 'Philips' and 50 electric bulbs of 'HMT'. He finds that 'Philips' bulbs gave an average life of 1500 hours with a standard deviation of 60 hours and 'HMT' bulbs gave an average life of 1512 hours with a standard deviation of 80 hours. Is there a significant difference in the mean life of the two makes of bulbs?

[MD Univ., MCom, 1998; Kumaon Univ., MBA, 1999]

10.8 Consider the following hypothesis:

$$H_0 : \mu = 15 \text{ and } H_1 : \mu \neq 15$$

A sample of 50 provided a sample mean of 14.2 and standard deviation of 5. Compute the p-value, and conclude about H_0 at the evel of significance 0.02.

10.9 A product is manufactured in two ways. A pilot test on 64 items from each method indicates that the products of method 1 have a sample mean tensile strength of 106 lbs and a standard deviation of 12 lbs, whereas in method 2 the corresponding values of mean and standard deviation are 100 lbs and 10 lbs, respectively. Greater tensile strength in the product is preferable. Use an appropriate large sample test of 5 per cent level of significance to test whether or not method 1 is better for processing the product. State clearly the null hypothesis. *[Delhi Univ., MBA, 2003]*

10.10 Two types of new cars produced in India are tested for petrol mileage. One group consisting of 36 cars averaged 14 kms per litre. While the other group consisting of 72 cars averaged 12.5 kms per litre.

(a) What test-statistic is appropriate if $\sigma_1^2 = 1.5$ and $\sigma_2^2 = 2.0$?

(b) Test whether there exists a significant difference in the petrol consumption of these two types of cars. (use $\alpha = 0.01$) *[Roorkee Univ., MBA, 2000]*

Hints and Answers

10.1 Let $H_0 : \mu = 1800$ and $H_1 : \mu \neq 1800$ (Two-tailed test)

Given $\bar{x} = 1850$, $n = 50$, $\sigma = 100$, $z_\alpha = \pm 2.58$ at $\alpha = 0.01$ level of significance

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{1850 - 1800}{100/\sqrt{50}} = 3.54$$

Since z_{cal} (= 3.54) > z_α (= 2.58), reject H_0. The breaking strength of the cables of 1800 does not support the claim.

10.2 Given $n = 100$, $\sigma = 50$; $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{50}{\sqrt{100}} = 5.$

Confidence interval at 99% is: $\bar{x} \pm z_\alpha \sigma_{\bar{x}} = 628 \pm 2.58$ (5) = 628 ± 12.9; $615.1 \leq \mu \leq 640.9$

Since hypothesized population mean $\mu = 640$ lies in the this interval, H_0 is accepted.

10.3 Let $H_0 : \mu = 1.28$ and $H_1 : \mu < 1.28$ (One-tailed test)

Given $n = 40$, $\bar{x} = 1.08$, $s = 0.05$, $z_\alpha = \pm 1.645$ at $\alpha = 0.05$ level of significance

$$z = \frac{\bar{x} - \mu}{s\sqrt{n}} = \frac{1.08 - 1.28}{0.05/\sqrt{40}} = -28.57$$

Since z_{cal} (= – 28.57) < $z_{\alpha/2}$ = – 1.64, H_0 is rejected. Mean life of the boots is less than 1.28 and affected by use in the desert.

10.4 Let $H_0 : \mu = 8.9$ and $H_1 : \mu \neq 8.9$ (Two-tail test)

Given $n = 50$, $\bar{x} = 9.3$, $s = 1.8$, $z_{\alpha/2} = \pm 1.96$ at $\alpha = 0.05$ level of significance

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{9.3 - 8.9}{1.8/\sqrt{50}} = 1.574$$

Since z_{cal} (= 1.574) < $z_{\alpha/2}$ (= 1.96), H_0 is accepted, that is, claim is valid.

10.5 Let $H_0 : \mu = 40{,}000$ and $H_1 : \mu \neq 40{,}000$ (Two-tail test)

Given $n = 100$, $\bar{x} = 39{,}350$, $s = 3{,}260$, and $z_{\alpha/2} = \pm 2.58$ at $\alpha = 0.01$ level of significance

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{39{,}350 - 40{,}000}{3260/\sqrt{100}} = -1.994$$

Since z_{cal} (= –1.994) > $z_{\alpha/2}$ (= – 2.58), H_0 is accepted. Thus the difference in the mean life of the tyres could be due to sampling error.

10.6 Let $H_0 : \mu_1 = \mu_2$ and $H_1 : \mu_1 > \mu_2$; μ_1 and μ_2 = mean height of Austrians and Englishmen, respectively.

Given, Austrian : $n_1 = 1600$, $\bar{x}_1 = 68.55$, $s_1 = 2.52$ and Englishmen; $n_2 = 6400$, $\bar{x}_2 = 67.85$, $s_2 = 2.56$

$$z = \frac{x_1 - x_2}{\sigma_{\bar{x}_1 - \bar{x}_2}} = \frac{x_1 - x_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{68.55 - 67.85}{\sqrt{\frac{(2.52)^2}{1600} + \frac{(2.56)^2}{6400}}} = 9.9$$

Since z_{cal} = 9.9 > z_α (= 2.58) for right tail test, H_0 is rejected. Austrian's are on the average taller than the Englishmen.

10.7 Let $H_0 : \mu_1 = \mu_2$ and $H_1 : \mu_1 \neq \mu_2$; μ_1 and μ_2 = mean life of Philips and HMT electric bulbs, respectively

Given, Philips : $n_1 = 50$, $\bar{x}_1 = 1500$, $s_1 = 60$ and HMT: $n_2 = 50$, $\bar{x}_2 = 1512$, $s_2 = 80$

$$z = \frac{x_1 - x_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{1500 - 1512}{\sqrt{\frac{(60)^2}{50} + \frac{(80)^2}{50}}}$$

$$= -\frac{12}{14.14} = -0.848$$

Since z_{cal} (= – 0.848) > $z_{\alpha/2}$ (= – 2.58) at $\alpha = 0.01$ level of significance, H_0 is accepted. Mean life of the two makes is almost the same, difference (if any) is due to sampling error.

10.8 Given $n = 50$, $\bar{x} = 14.2$, $s = 5$, and $\alpha = 0.02$

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{14.2 - 15}{5/\sqrt{50}} = -1.13$$

Table value of z = 1.13 is 0.3708. Thus p-value = 2 (0.5000 – 0.3708) = 0.2584. Since p-value > α, H_0 is accepted.

10.9 Let $H_0 : \mu_1 = \mu_2$ and $H_1 : \mu_1 > \mu_2$; μ_1 and μ_2 = mean life of items produced by Method 1 and 2, respectively.

Given, Method 1: $n_1 = 64$, $\bar{x}_1 = 106$, $s_1 = 12$; Method 2: $n_2 = 64$, $\bar{x}_2 = 100$, $s_2 = 10$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{106 - 100}{\sqrt{\frac{(12)^2}{64} + \frac{(10)^2}{64}}} = 3.07$$

Since z_{cal} (= 3.07) > z_α (= 1.645) for a right-tailed test, H_0 is rejected. Method 1 is better than Method 2.

10.10 Let $H_0 : \mu_1 = \mu_2$ and $H_1 : \mu_1 \neq \mu_2$; μ_1 and μ_2 = mean petrol mileage of two types of new cars, respectively

Given $n_1 = 36$, $\bar{x}_1 = 14$, $\sigma_1^2 = 1.5$ and $n_2 = 72$, $\bar{x}_2 = 12.5$, $\sigma_2^2 = 2.0$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{14 - 12.5}{\sqrt{\frac{1.5}{36} + \frac{2}{72}}}$$

$$= \frac{1.5}{0.2623} = 5.703$$

Since z_{cal} (= 5.703) > $z_{\alpha/2}$ (= 2.58) at $\alpha = 0.01$ level of significance, H_0 is rejected. There is a significant difference in petrol consumption of the two types of new cars.

10.8 HYPOTHESIS TESTING FOR SINGLE POPULATION PROPORTION

Sometimes instead of testing a hypothesis pertaining to a population mean, a population proportion (a fraction, ratio or percentage) p of values that indicates the part of the population or sample having a particular attribute of interest is considered. For this, a random sample of size n is selected to compute the proportion of elements having a particular attribute of interest (also called success) in it as follows:

$$\bar{p} = \frac{\text{Number of successes in the sample}}{\text{Sample size}} = \frac{x}{n}$$

The value of this statistic is compared with a hypothesized population proportion p_0 so as to arrive at a conclusion about the hypothesis.

The three forms of null hypothesis and alternative hypothesis pertaining to the hypothesized population proportion p are as follows:

Null hypothesis	*Alternative hypothesis*
• $H_0 : p = p_0$	$H_1 : p \neq p_0$ (Two-tailed test)
• $H_0 : p \geq p_0$	$H_1 : p < p_0$ (Left-tailed test)
• $H_0 : p \leq p_0$	$H_1 : p > p_0$ (Right-tailed test)

To conduct a test of a hypothesis, it is assumed that the sampling distribution of a proportion follows a standardized normal distribution. Then, using the value of the sample proportion $\bar{p}$ and its standard deviation $\sigma_{\bar{p}}$, we compute a value for the z-test statistic as follows:

$$\text{Test statistic } z = \frac{\bar{p} - p_0}{\sigma_{\bar{p}}} = \frac{\bar{p} - p_0}{\sqrt{\dfrac{p_0(1 - p_0)}{n}}}$$

The comparison of the z-test statistic value to its critical (table) value at a given level of significance enables us to test the null hypothesis about a population proportion based on the difference between the sample proportion $\bar{p}$ and the hypothesized population proportion.

Decision rule: Reject H_0 when

One-tailed test	*Two-tailed test*
• $z_{cal} > z_\alpha$ or $z_{cal} < -z_\alpha$ when $H_1 : p < p_0$	• $z_{cal} > z_{\alpha/2}$ or $z_{cal} < -z_{\alpha/2}$
• p-value $< \alpha$	

10.8.1 Hypothesis Testing for Difference Between Two Population Proportions

Let two independent populations each having proportion and standard deviation of an attribute be as follows:

Population	*Proportion*	*Standard Deviation*
1	p_1	σ_{p_1}
2	p_2	σ_{p_2}

The hypothesis testing concepts developed in the previous section can be extended to test whether there is any difference between the proportions of these populations. The null hypothesis that there is no difference between two population proportions is stated as:

$$H_0 : p_1 = p_2 \text{ or } p_1 - p_2 = 0 \quad \text{and} \quad H_1 : p_1 \neq p_2$$

The sampling distribution of difference in sample proportions $\bar{p} - \bar{p}_2$ is based on the assumption that the difference between two population proportions, $p_1 - p_2$ is normally distributed. The standard deviation (or error) of sampling distribution of $p_1 - p_2$ is given by

$$\sigma_{\bar{p}_1-\bar{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1}+\frac{p_2(1-p_2)}{n_2}}\ ;\ q_1 = 1-p_1 \text{ and } q_2 = 1-p_2$$

where the difference $\bar{p}_1 - \bar{p}_2$ between sample proportions of two independent simple random samples is the point estimator of the difference between two population proportions. Obviously expected value, $E(\bar{p}_1 - \bar{p}_2) = p_1 - p_2$.

Thus the z-test statistic for the difference between two population proportions is stated as:

$$z = \frac{(\bar{p}_1-\bar{p}_2)-(p_1-p_2)}{\sigma_{\bar{p}_1-\bar{p}_2}} = \frac{\bar{p}_1-\bar{p}_2}{\sigma_{\bar{p}_1-\bar{p}_2}}$$

Invariably, the standard error $\sigma_{\bar{p}_1-\bar{p}_2}$ of difference between sample proportions is not known. Thus when a null hypothesis states that there is no difference between the population proportions, we combine two sample proportions $\bar{p}_1$ and $\bar{p}_2$ to get one unbiased estimate of population proportion as follows:

$$\text{Pooled estimate } \bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1+n_2}$$

The z-test statistic is then restated as:

$$z = \frac{\bar{p}_1-\bar{p}_2}{s_{\bar{p}_1-\bar{p}_2}}\ ;\ s_{\bar{p}_1-\bar{p}_2} = \sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$

Example 10.12: An auditor claims that 10 per cent of customers' ledger accounts are carrying mistakes of posting and balancing. A random sample of 600 was taken to test the accuracy of posting and balancing and 45 mistakes were found. Are these sample results consistent with the claim of the auditor? Use 5 per cent level of significance.

Solution: Let us take the null hypothesis that the claim of the auditor is valid, that is,

$$H_0 : p = 0.10 \quad \text{and} \quad H_1 : p \neq 0.10 \quad \text{(Two-tailed test)}$$

Given $\bar{p}$ = 45/600 = 0.075, n = 600, and α = 5 per cent. Thus using the z-test statistic

$$z = \frac{\bar{p}-p_0}{\sigma_{\bar{p}}} = \frac{0.075-0.10}{\sqrt{\frac{0.10\times 0.90}{600}}} = -\frac{0.025}{0.0122} = -2.049$$

Since z_{cal} (= – 2.049) is less than its critical (table) value z_α (= – 1.96) at α = 0.05 level of significance, null hypothesis, H_0 is rejected. Hence, we conclude that the claim of the auditor is not valid.

Example 10.13: A manufacturer claims that at least 95 per cent of the equipments which he supplied to a factory conformed to the specification. An examination of the sample of 200 pieces of equipment revealed that 18 were faulty. Test the claim of the manufacturer.

Solution: Let us take the null hypothesis that at least 95 per cent of the equipments supplied conformed to the specification, that is,

$$H_0 : p \geq 0.95 \quad \text{and} \quad H_1 : p < 0.95 \text{ (Left-tailed test)}$$

Given $\bar{p}$ = per cent of pieces conforming the specification = 1 – (18/100) = 0.91

n = 200 and level of significance α = 05. Thus using the z-test statistic,

$$z = \frac{\bar{p}-p_0}{\sigma_{\bar{p}}} = \frac{0.91-0.95}{\sqrt{\frac{0.95\times 0.05}{200}}} = -\frac{0.04}{0.015} = -2.67$$

Since z_{cal} (= – 2.67) is less than its critical value z_α (= – 1.645) at α = 0.05 level of significance, the null hypothesis, H_0 is rejected. Hence we conclude that the proportion of equipments conforming to specifications is not 95 per cent.

Example 10.14: A company is considering two different television advertisements for promotion of a new product. Management believes that advertisement A is more effective than advertisement B. Two test market areas with virtually identical consumer characteristics are selected: advertisement A is used in one area and advertisement B in the other area. In a random sample of 60 customers who saw advertisement A, 18 had tried the product. In a random sample of 100 customers who saw advertisement B, 22 had tried the product. Does this indicate that advertisement A is more effective than advertisement B, if a 5 per cent level of significance is used? *[Delhi Univ., MFC 1996; MBA, 2000]*

Solution: Let us take the null hypothesis that both advertisements are equally effective, that is,

$$H_0 : p_1 = p_2 \quad \text{and} \quad H_1 : p_1 > p_2 \text{ (Right-tailed test)}$$

where p_1 and p_2 = proportion of customers who saw advertisement A and advertisement B respectively.

Given $n_1 = 60$, $\bar{p}_1 = 18/60 = 0.30$; $n_2 = 100$, $\bar{p}_2 = 22/100 = 0.22$ and level of significance $\alpha = 0.05$. Thus using the z-test statistic.

$$z = \frac{(\bar{p}_1 - \bar{p}_2) - (p_1 - p_2)}{s_{\bar{p}_1 - \bar{p}_2}} = \frac{\bar{p}_1 - \bar{p}_2}{s_{\bar{p}_1 - \bar{p}_2}}; \quad p_1 = p_2$$

where
$$s_{\bar{p}_1 - \bar{p}_2} = \sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}; \quad q = 1 - p$$

$$= \sqrt{0.25 \times 0.75\left(\frac{1}{60} + \frac{1}{100}\right)} = \sqrt{0.1875\left(\frac{160}{600}\right)} = 0.0707;$$

$$\bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2} = \frac{60\,(18/60) + 100\,(22/100)}{60 + 100}$$

$$= \frac{18 + 22}{160} = \frac{40}{160} = 0.25$$

Substituting values in z-test statistic, we have

$$z = \frac{0.30 - 0.22}{0.0707} = \frac{0.08}{0.0707} = 1.131$$

Since $z_{cal} = 1.131$ is less than its critical value $z_\alpha = 1.645$ at $\alpha = 0.05$ level of significance, the null hypothesis, H_0 is accepted. Hence we conclude that there is no significant difference in the effectiveness of the two advertisements.

Example 10.15: In a simple random sample of 600 men taken from a big city, 400 are found to be smokers. In another simple random sample of 900 men taken from another city 450 are smokers. Do the data indicate that there is a significant difference in the habit of smoking in the two cities? *[Raj Univ., MCom, 1998; Punjab Univ., MCom, 1996]*

Solution: Let us take the null hypothesis that there is no significant difference in the habit of smoking in the two cities, that is,

$$H_0 : p_1 = p_2 \quad \text{and} \quad H_1 : p_1 \neq p_2 \text{ (Two-tailed test)}$$

where p_1 and p_2 = proportion of men found to be smokers in the two cities.

Given, $n_1 = 600$, $\bar{p}_1 = 400/600 = 0.667$; $n_2 = 900$, $\bar{p}_2 = 450/900 = 0.50$ and level of significance $\alpha = 0.05$. Thus using the z-test statistic

$$z = \frac{(\bar{p}_1 - \bar{p}_2) - (p_1 - p_2)}{s_{\bar{p}_1 - \bar{p}_2}} = \frac{\bar{p}_1 - \bar{p}_2}{s_{\bar{p}_1 - \bar{p}_2}}; \quad p_1 = p_2$$

where
$$s_{\bar{p}_1 - \bar{p}_2} = \sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}; \quad q = 1 - p$$

$$= \sqrt{0.567 \times 0.433\left(\frac{1}{600} + \frac{1}{900}\right)} = \sqrt{0.245\,(0.002)} = 0.026;$$

$$\bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2} = \frac{600\,(400/600) + 900\,(450/900)}{600 + 900}$$

$$= \frac{400 + 450}{1500} = \frac{850}{1500} = 0.567$$

Substituting values in z-test statistic, we have

$$z = \frac{0.667 - 0.500}{0.026} = \frac{0.167}{0.026} = 6.423$$

Since $z_{cal} = 6.423$ is greater than its critical value $z_{\alpha/2} = 2.58$, at $\alpha/2 = 0.025$ level of significance, the null hypothesis, H_0 is rejected. Hence we conclude that there is a significant difference in the habit of smoking in two cities.

10.9 HYPOTHESIS TESTING FOR A BINOMIAL PROPORTION

The sampling of traits or attributes is considered as drawing of samples from a population whose elements have a particular trait of interest. For example, in the study of attribute such 'good or acceptable' pieces of items manufactured by company, a sample of suitable size may be taken from a given lot of items to classify them as good or not.

Instead of examining the hypothesis regarding *proportion of elements having the same trait (called sucess)* in a sample as discussed in the previous section, we could examine the *number of successes* in a sample. The z-test statistic for determining the magnitude of the difference between the number of successes in a sample and the hypothesized (expected) number of successes in the population is given by

$$z = \frac{\text{Sample estimate} - \text{Expected value}}{\text{Standard error of estimate}} = \frac{x - np}{\sqrt{npq}}$$

Recalling from previous discussion that although sampling distribution of the number of successes in the sample follows a binomial distribution having its mean $\mu = np$ and standard deviation $\sqrt{npq}$, the normal distribution provides a good approximation to the binomial distribution provided the sample size is large, that is, both $np \geq 5$ and $n\ (1 - p) \geq 5$.

Example 10.16: Suppose the production manager implements a newly developed sealing system for boxes. He takes a random sample of 200 boxes from the daily output and finds that 12 need rework. He is interested to determine whether the new sealing system has increased defective packages below 10 per cent. Use 1 per cent level of significance

Solution: Let us state the null and alternative hypotheses as follows:

$$H_0 : p \geq 0.10 \quad \text{and} \quad H_1 : p < 0.10 \text{ (Left-tailed test)}$$

Given $n = 200, p = 0.10$, and level of significance $\alpha = 0.01$. Applying the z-test statistic

$$z = \frac{x - np}{\sqrt{npq}} = \frac{12 - 200\,(0.10)}{\sqrt{200\,(0.10)\,(0.90)}} = \frac{12 - 20}{\sqrt{18}} = -1.885$$

Since z_{cal} (–1.885) is more than its critical value $z_{\alpha} = -2.33$ for one-tailed test at $\alpha = 0.01$ level of significance, the null hypothesis, H_0 is accepted. Hence we conclude that the proportion of defective packages with the new sealing system is more than 10 per cent.

Example 10.17: In 324 throws of a six-faced dice, odd points appeared 180 times. Would you say that the dice is fair at 5 per cent level of significance?

[*MD Univ., MCom, 1997*]

Solution: Let us take the hypothesis that the dice is fair, that is,

$$H_0 : p = 162/324 = 0.5 \quad \text{and} \quad H_1 : p \neq 0.5 \text{ (Two-tailed test)}$$

Given $n = 324, p = q = 0.5$ (i.e., 162 odd or even points out of 324 throws). Applying the z-test statistic:

$$z = \frac{x - np}{\sqrt{npq}} = \frac{180 - 162}{\sqrt{324 \times 0.5 \times 0.5}} = \frac{18}{9} = 2$$

Since $z_{cal} = 2$ is more than its critical value $z_{\alpha/2} = 1.96$ at $\alpha/2 = 0.025$ significance level, the null hypothesis H_0 is rejected. Hence we conclude that the dice is not fair.